#### GIRLS' HIGH SCHOOL AND COLLEGE

2020 - 2021

**CLASS - 11 A&B** 

**PHYSICS** 

WORKSHEET- 02

Chapter- DIMENSIONAL ANALYSIS

Topic - DIMENSIONAL ANALYSIS

<u>INSTRUCTIONS</u>: Parents kindly instruct you ward to visit the relevant websites or refer <u>Nootan ISC 11 Physics- 11 by Kumar & Mittal</u> (Nageen Prakashan) or <u>Physics - 11 by DK Tyagi</u>( Balaji Publications) to answer the following questions on the given topic.

NOTE: The child should go through the subject matter thoroughly before answering the questions that follow.

The topic to be discussed in this worksheet is a very basic and an important topic in Physics so one needs to pay a lot of attention to it.

When we study about any quantity in Physics we analyze it by its measured value which includes a magnitude and its unit.

We also know that units are fundamental and derived. To measure any quantity it is ultimately the Fundamental Quantity/ Quantities that have to be measured. The aspect of measurement of these quantities is called a DIMENSION.

When we analyze that, to measure a given physical quantity- how many such dimensions have to be measured and that too how many times then such a process is called DIMENSIONAL ANALYSIS. Writing a quantity in terms of its dimensions and the number of times they are measured is called a DIMENSIONAL FORMULA.( [MaLbTc])

Eg. Speed is a quantity that measures distance (i.e. length) and time both once each. Hence its dimensional formula will be LT<sup>-1</sup> (L for length

and T for time).

Similarly for acceleration the corresponding formula will be LT<sup>-2</sup> (ms<sup>-2</sup>). Now study the following subject matter carefully.

# Dimensional Analysis

#### Syllabus

Dimensional formulae of physical quantities and constants, dimensional analysis and its applications.

#### Dimensions of Physical Quantities

The unit of a physical quantity can be written in different ways. For example, velocity can be expressed in metre/second, km/hour or km/minute, but in every case we divide the unit of length by the unit of time, that is,

unit of velocity = 
$$\frac{\text{unit of length}}{\text{unit of time}}$$
 =  $(\text{unit of length})^1 \times (\text{unit of time})^{-1}$ .

Thus, in order to get the unit of velocity, we raise the unit of length to the power 1 and the unit of time to the power -1. These powers are called the 'dimensions of velocity'. In other words, the dimensions of velocity are 1 in length and -1 in time. Thus, the dimensions of a physical quantity are the powers to which the fundamental units are raised in order to obtain the derived unit of that quantity.

To express the dimensions of physical quantities in mechanics, the length, mass and time are denoted by [L], [M] and [T]. If the dimensions of a physical quantity are a in length, b in mass and c in time, then the dimensions of that physical quantity shall be written in the following manner:

 $[L^a M^b T^c].$ 

This is called the 'dimensional formula' of the quantity.

#### Dimensional Formulae of Some Physical Quantities

In order to get dimensional formula of a physical quantity, the quantity is described in terms of other simple quantities of known dimensions. Dimensional formulae of certain physical quantities and their SI units are given below:

S. No.	Physical Quantity	Relation with Other Physical Quantities	Dimensional Formula	SI Unit
1.	Area	length × breadth	$[L \times L] = [L^2] = [M^0 L^2 T^0]$	$m^2$
2.	Volume	length × breadth × height	$[\mathbf{L}\times\mathbf{L}\times\mathbf{L}]=[\mathbf{L}^3]=[\mathbf{M}^0\mathbf{L}^3\mathbf{T}^0]$	m³
3.	Density	mass volume	$\frac{[M]}{[L^3]} = \{M \ L^{-3}\} = \{M \ L^{-3} \ T^0\}$	kg m <sup>-3</sup>
4.	Velocity (or Speed)	displacement (or distance) time	$\frac{\{L\}}{\{T\}} \approx \{L \ T^{-1}\} \approx \{M^0 \ L \ T^{-1}\}$	m s <sup>-1</sup>
5.	Acceleration	change in velocity	$\frac{[L T^{-1}]}{[T]} \approx [L T^{-2}] = [M^0 L T^{-2}]$	$m s^{-2}$
5.	Force	mass × acceleration	$[M] [LT^2] = [MLT^2]$	kg m s <sup>-2</sup> or N (newton

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5	. Work	force × displacement	$[M L T^{2}] [L] = [ML^{2} T^{2}]$	kg m <sup>2</sup> s <sup>-2</sup> or J (joule)
8	Acceleration due to Gravity g	weight (force) mass	$\frac{[M L T^{-2}]}{[M]} = [M^0 L T^{-2}]$	m s <sup>-2</sup>
9.	Power	work time	$\frac{[M L^2 T^{-2}]}{[T]} = [ML^2 T^{-3}]$	or M (MSH)
10.	Linear Momentum	mass × velocity	$[M] [L T^{-1}] = [MLT^{-1}]$	kg m s <sup>-1</sup>
11.	Kinetic Energy	$\frac{1}{2} \times \text{mass} \times (\text{velocity})^2$	[M] $[L T^{-1}]^2 = [M L^2 T^{-2}]$	J
12.	Potential Energy	mass × acceleration due to gravity × height	[M] [L $T^{-2}$ ] [L] = [M $L^2 T^{-2}$ ]	J
13.	Pressure	force area	$\frac{[M L T^{-2}]}{[L^2]} = [M L^{-1} T^{-2}]$	N m <sup>-2</sup>
14.	Impulse	force × time	$[MLT^{-2}][T] = [MLT^{-1}]$	Ns
15.	Moment of Force		$[M L T^{-2}] [L] = [M L^2 T^{-2}]$	Nm
1.5.	Moment of Porce	force × force arm	[MLI][L] = [MLI]	.,
16.	Force Constant of Spring	applied force extension in length	$\frac{[M L T^{-2}]}{[L]} = [M L^0 T^{-2}]$	N m <sup>-1</sup>
		force	(MI T <sup>-2</sup> )	
17.	Stress	area	$\frac{[M L T^{-2}]}{[T^2]} = [M L^{-1} T^{-2}]$	N m <sup>-2</sup>
			[L ]	
18.	Strain	extension in length original length	$\frac{[L]}{[L]} = [L^0] = [M^0 L^0 T^0]$	no unit
19.	Coefficient of		c 1 7-	
	Elasticity	stress	$\frac{[M L^{-1} T^{-2}]}{\Pi^{0}} = [M L^{-1} T^{-2}]$	$N m^{-2}$
		strain	[r <sub>0</sub> ]	74 111
		<b>C</b>		
20.	Surface Tension	force	$\frac{[M L T^{-2}]}{[L]} = [M L^0 T^{-2}]$	$N m^{-1}$
		length	[L] - [M.F. 1 ]	IV III
		Water-parks		
21.	Surface Energy	energy	$\frac{[ML^2T^{-2}]}{[I^2]} = [ML^0T^{-2}]$	$\mathrm{J}\mathrm{m}^{-2}$
		area	[L <sup>2</sup> ]	Jiii
		change in velocity		
22.	Velocity Gradient	perp. distance	$\frac{[L T^{-1}]}{[1]} = [M^0 L^0 T^{-1}]$	$s^{-1}$
		perp. distance	[r] - fw r 1	3
23.	Coefficient of	force		
	Viscosity	area × velocity gradient	$\frac{[M L T^{-2}]}{[L^2] [T^{-1}]} = [M L^{-1} T^{-1}]$	kg m <sup>-1</sup> s <sup>-1</sup>
24	<b>52</b> 0 0 5	aca - relocity gradient	$[L^2][T^{-1}]$	kg iii s
24.	Gravitational Constant G	$F = G \frac{m_1 m_2}{r^2}$		
		F -2		
		$\therefore G = \frac{Fr^2}{m_1 m_2}$		
			(M) T-2, m2,	
		$= \frac{\text{force} \times (\text{distance})^2}{\text{mass} \times \text{mass}}.$	$\frac{[M L T^{-2}] [L^2]}{[M] [M]} = [M^{-1} L^3 T^{-2}]$	kg <sup>-1</sup> m <sup>3</sup> s <sup>-2</sup> or N m <sup>2</sup> kg <sup>-2</sup>

Pure ratios and pure numbers do not have any dimensions. Such quantities are called 'dimensionless'. Angle, strain, relative density,  $\pi$ , sin  $\theta$ , etc., are dimensionless quantities.

frequency

 $\frac{[M L^2 T^{-2}]}{[T^{-1}]} = [M L^2 T^{-1}] \qquad \text{kg m}^2 s^{-1}$ 

41.

Planck's Constant h Knowing the dimensional formula of a physical quantity, we can write its units. For example, the dimensional formula for G (gravitational constant) is  $[M^{-1} L^3 T^{-2}]$ . From this we can write the unit of G as  $kg^{-1} m^3 s^{-2}$ .

### 3 Classification of Quantities

On the basis of dimensional analysis, quantities can be put in four groups as follows:

- (i) Dimensional Variables: These quantities possess dimensions and are liable to variation e.g., force velocity, work, momentum, heat etc.
- (ii) Non-dimensional Variables: These quantities do not have dimensions but are liable to variations e.g., strain, angle, Poisson's ratio, relative density etc.
- (III) Dimensional Constants: These quantities possess dimensions but are not liable to variations e.g. Planck's constant, Boltzmann constant, gravitational constant etc.
- (Iv) Non-dimensional Constants: These quantities do not possess any dimensions and do not change e.g. Joule's constant, pure numbers (1, 2, 3, ...) etc.

Note: A dimensionless quantity may have unit (e.g., J, angle etc.) but a unitless quantity cannot possess dimensions.

# Principle of Homogeneity

It states, "every equation relating physical quantities must be dimensionally homogeneous", i.e., the dimensions of each term of a physical equation must be the same.

The principle in other words, implies that only those quantities can be added, subtracted or equated which have the same dimensions. For example, if we have the equation

$$Q = W + X$$

where Q is the heat energy, W is the work and X is some unknown quantity, then dimensions of Q, Wand X must be the same. We know that dimensional formula of Q and W is  $[ML^2T^{-2}]$ , hence dimensional formula of unknown quantity X must also be  $[ML^2T^{-2}]$ .

## Uses of Dimensional Equations

1. To Convert Units of One System into the Units of Other System: The product of the numerical value of a physical quantity and its corresponding unit is a constant. For example, if the numerical values of a physical quantity p are  $n_1$  and  $n_2$  in two different systems and the corresponding units are  $u_1$  and  $u_2$ ,

$$p = n_1 (u_1) = n_2 (u_2)$$
 ...(

Now, if the dimensions of the physical quantity are a in mass, b in length and c in time, then its dimensional formula will be  $[M^a L^b T^c]$ . If the fundamental units in one system are  $M_1$ ,  $L_1$  and  $T_1$ , then

$$p = n_1 (M_1^a L_1^b T_1^c).$$

Similarly, if the fundamental units in the second system are  $M_2$ ,  $L_2$  and  $T_2$ , then  $p=n_2 \left(M_2^a L_2^b T_2^c\right)$ .

$$p = n_2 (M_2^a L_2^b T_2^c).$$

According to eq. (i), we have

$$n_{1} (M_{1}^{a} L_{1}^{b} T_{1}^{c}) = n_{2} (M_{2}^{a} L_{2}^{b} T_{2}^{c}).$$

$$\therefore n_{2} = n_{1} \left(\frac{M_{1}}{M_{2}}\right)^{a} \left(\frac{L_{1}}{L_{2}}\right)^{b} \left(\frac{T_{1}}{T_{2}}\right)^{c}.$$
...(ii)

Using this formula, we can convert the numerical value of a physical quantity from one system of units into the other system.

Example: We consider the physical quantity 'force'. Its unit in MKS (or SI) system is 'newton' and in CGS system is 'dyne'. Let us convert 1 newton into dynes.

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The dimensional formula of force is  $[M L T^{-2}]$ . Suppose,  $M_1$ ,  $L_1$ ,  $T_1$  represent kilogram (kg), metre (m), second (s) and  $M_2$ ,  $L_2$ ,  $T_2$  represent gram (g), centimetre (cm), second (s) respectively. Then, the units of force in MKS and CGS systems will be  $(M_1L_1T_1^{-2})$  and  $(M_2 L_2 T_2^{-2})$  respectively. If the numerical values of force are  $n_1$  and  $n_2$  respectively, then

very, then  $n_1 (M_1 L_1 T_1^{-2}) = n_2 (M_2 L_2 T_2^{-2})$   $n_2 = n_1 \left(\frac{M_1}{M_2}\right) \left(\frac{L_1}{L_2}\right) \left(\frac{T_1}{T_2}\right)^{-2}.$ 

Here  $n_1 = 1$ .

or

 $\therefore n_2 = 1 \left(\frac{kg}{g}\right) \left(\frac{m}{cm}\right) \left(\frac{s}{s}\right)^{-2}$   $= 1 \left(\frac{10^3 g}{g}\right) \left(\frac{10^2 cm}{cm}\right) \left(\frac{s}{s}\right)^{-2} = 1 \times 10^3 \times 10^2 \times 1 = 10^5.$ 

Thus,

1 newton =  $10^5$  dynes.

2. To Check the Correctness of an Equation: The principle of homogeneity enables us to check the correctness of a physical equation. For this, we examine the dimensions of the left-hand side and the right-hand side of the equation. If the dimensions of both the sides are same, then the equation is correct, otherwise not.

**Example:** Suppose, we have to check the correctness of the equation  $\frac{1}{2}mv^2 = mgh$ , where m is the mass

of a body, v its velocity, g is acceleration due to gravity and h is the height. The dimensional formulae of the various quantities in the equation are :

 $\begin{aligned} \text{mass, } m &= [\mathbf{M}] \\ \text{velocity, } v &= [\mathbf{L}\mathbf{T}^{-1}] \\ \text{acceleration due to gravity, } g &= [\mathbf{L}\mathbf{T}^{-2}] \\ \text{height, } h &= [\mathbf{L}]. \end{aligned}$ 

 $\frac{1}{2}$  is a pure ratio, having no dimensions. Substituting these dimensional formulae in the equation

 $\frac{1}{2}mv^2 = mgh$ , we have

[M]  $[LT^{-1}]^2 = [M] [LT^{-2}] [L]$  $[ML^2T^{-2}] = [ML^2T^{-2}].$ 

or

The dimensions on both the sides are same. Hence, the given equation is correct.

- 3. To Establish the Relation Among Various Physical Quantities: If we know the factors on which a given physical quantity may possibly depend, then, using dimensions, we can find a formula relating the quantity with those factors. Let us take a few examples:
- (i) Expression for the Time-period of a Simple Pendulum: The time-period T of a simple pendulum may depend on the following:

mass (m) of the bob, length (l) of the thread

and

acceleration due to gravity (g).

To establish the relation among these, let us suppose that the time-period T depends on mass raised to the power a, on length raised to the power b and on acceleration due to gravity raised to the power c. That is,

 $T \propto (m)^a (l)^b (g)^c$  $T = k (m)^a (l)^b (g)^c, \qquad \dots (i)$ 

or

where k is a dimensionless constant. Writing the dimensions of both the sides, we have

$$[T] = [M]^a [L]^b [LT^{-2}]^c$$

$$[M^0 L^0 T^1] = [M^a L^{b+c} T^{-2c}].$$

where k is a dimensionless constant. Writing the dimensions of vocat the sides, we have  $[T] = [M]^a [L]^b [LT^{-2}]^c$ or  $[M^0 L^0 T^1] = [M^a L^{b+c} T^{-2c}].$ By the principle of homogeneity of dimensions, the dimensions on the two sides of this equation must be the same. That is

$$a = 0$$
$$b + c = 0$$

and

$$-2c=1.$$

Solving these three equations, we get

$$a = 0$$
,  $c = -\frac{1}{2}$  and  $b = \frac{1}{2}$ .

Putting these values in eq. (i), we get

$$T = k (m)^0 (l)^{1/2} (g)^{-1/2}$$

or

$$T = k \sqrt{\frac{l}{g}}.$$

This is the formula for the period of a simple pendulum. It is clear that time-period does not depend upon the mass of the bob. From the dimensional equation, the value of k cannot be known. However, on the basis of experiments the value of  $k = 2\pi$ .

$$\therefore T = 2\pi \sqrt{\frac{l}{g}}.$$

(ii) Frequency of a Stretched String: The frequency (n) of a stretched string depends upon the tension F applied, length l of the string and the mass per unit length m of the string. To establish the relation among them, suppose that the frequency n depends on the tension raised to the power a, length raised to the power b and mass per unit length raised to the power c. That is,

 $n \propto (F)^a (l)^b (m)^c$ 

or

$$n = k (F)^{a} (I)^{b} (m)^{c} \qquad \dots (i)$$

where k is a dimensionless constant. Writing the dimensions of both the sides:

$$[T^{-1}] = [M L T^{-2}]^a [L]^b [M L^{-1}]^c$$
  
 $[M^0 L^0 T^{-1}] = [M^{a+c} L^{a+b-c} T^{-2a}].$ 

or

For dimensional balance, the dimensions on both sides should be same. Thus

$$a+c=0$$
$$a+b-c=0$$

and

$$a+b-c=0$$
$$-2a=-1.$$

Solving these three equations, we get

and

$$a = \frac{1}{2}$$
,  $c = -\frac{1}{2}$  and  $b = -1$ .

Substituting these values in eq. (i), we get

$$n = k (F)^{1/2} (l)^{-1} (m)^{-1/2}$$

$$n = \frac{k}{l} \sqrt{\frac{F}{m}}.$$

Experimentally, the value of  $k = \frac{1}{2}$ .

$$\therefore n = \frac{1}{2l} \sqrt{\frac{F}{m}}.$$

#### Limitations of Dimensional Analysis

The method of dimensions has the following limitations:

- (i) By this method, the value of any dimensionless constant involved in the formula cannot be calculated.
  (ii) If a physical quantity depends upon more than three factors, then relation among them cannot be established because we can have only three equations by equalising the powers of M, L and T and only the values of three unknowns can be calculated.
- (iii) By this method, the equation containing trigonometrical (sin 0, cos 0, ...) exponential and logarithmic terms cannot be analysed.
- (iv) Dimensional formula may not represent a unique physical quantity, hence physical quantities of different nature may provide dimensionally homogeneous equation e.g., work = torque.

# Solved NUMERICAL Problems

# I. Based on Checking the Correctness of an Equation and Principle of Homogeneity

**Example 1.** Check the correctness of the relation  $v^2 - v^2 = 2as$ , where v is the initial velocity of a particle and v is its final velocity after travelling a distance s under uniform acceleration a.

Solution. 
$$v^2 - u^2 = 2 \text{ as}$$
 ...(i)

The dimensional formulae of the various quantities in this equation are :

velocity, 
$$v$$
 or  $u = [LT^{-1}]$   
acceleration,  $a = [LT^{-2}]$   
distance,  $s = [L]$ .

The number 2 is dimensionless. Substituting these formula in the given equation (i), we have

$$[LT^{-1}]^2 - [LT^{-1}]^2 = [LT^{-2}][L]$$
  
 $[L^2T^{-2}] - [L^2T^{-2}] = [L^2T^{-2}].$ 

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The dimensions of each term on the left-hand side are the same as those or the right-hand side. Hence, the given relation is dimensionally correct.

Example 2. Check the correctness of the relation  $s = ut + \frac{1}{2}at^2$ , where u is the initial velocity of a

particle, a is the constant acceleration and s is the distance travelled in time t.

Solution. 
$$s = ut + \frac{1}{2}at^2.$$
 ...(i)

The dimensional formulae of the quantities occurring in the given relation are:

distance, 
$$s = [L]$$
  
velocity,  $u = [LT^{-1}]$   
time,  $t = [T]$   
acceleration,  $a = [LT^{-2}]$ .

Using these in eq. (i), we have (constant  $\frac{1}{2}$  is dimensionless)

$$[L] = [LT^{-1}][T] + [LT^{-2}][T^{2}]$$
  
=  $[L] + [L]$ .

The dimensions of each term on the right-hand side are the same as those of the term on the left-hand side. Hence, eq. (i) is dimensionally correct.

Now answer the following questions.

- Q1) Name all the basic dimensions.
- Q2) How will you express the dimensional formula for pressure?
- Q3) Name the physical quantity that has the same dimensional formula as frequency.
- Q4) Name two dimensionless quantities.
- Q5) Distinguish between dimensional and non- dimensional constants with example.
- Q6) If Electric Field is expressed as  $E = \frac{F}{q}$ , where 'F' is electric force and 'q' is electric charge, then what is the dimensional formula for E?
- Q7) If electric force F =  $k \frac{q_1 q_2}{r^2}$ , where  $q_1$ ,  $q_2$  are charges and r is distance between them then find the Dimensional Formula for 'k'.
- Q8) What is principle of homogeneity in equations?
- Q9) State the uses of Dimensional Analysis.
- Q10) Find the dimensional formula for Bulk Modulus 'B' using the expression  $v = \sqrt{\frac{B}{\rho}}$ ; v is velocity and  $\rho$  is density.
- Q11) The refractive index  $\mu = A + (\frac{B}{\lambda^2})$  where *A* and *B* are constants. Find their dimensional formula and SI units.
- Q12) Find the dimension of a/b in the relation  $F=a\sqrt{x}+bt^2$ , where 'F' is force,

'x' is distance and 't' is time.

Q13) Prove with the help of Dimensional Analysis that the equation h =  $\frac{1}{2}$  gt for distance travelled by a freely falling body under gravity in time

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't' is incorrect. Find the correct equation with the help of dimensions.

Q14) Check the authenticity of the equation  $t = k \sqrt{\frac{h}{g}}$  using Dimensional Analysis. Here t = time period; h = height of liquid column; g = acceleration due to gravity; k is a dimensionless constant.

- Q15) Given  $\rho = \frac{3g}{4\pi R_e G}$ ; where  $\rho$  is density,  $R_e$  is the radius of the earth an G gravitational constant. Check the correctness of this equation.
- Q16) Use Dimensional Analysis to check the correctness of the equation

S = ut + gt<sup>2</sup>. Is the relation actually correct? What limitation of Dimensional Analysis does it reveal?

- Q17) A particle of mass 'm' is tied to a string and swung around in a circular path of radius 'r' with a constant speed 'v'. Derive a formula for the centripetal force 'F' exerted by the particle on our hand, using the method of dimensions.
- Q18) The frequency 'n' of a tuning fork depends upon length 'l' of the prong, the density 'p' and Young's modulus 'Y' of its material. From dimensional considerations, find a possible formula for the frequency of the tuning fork.
- Q 19) It is observed that a liquid rises in a capillary to a certain height due to Surface Tension. This height 'h' to which a liquid rises in a capillary depends upon the radius f the capillary 'r', Surface Tension of liquid 'S', density of liquid ' $\rho$ ' and acceleration due to gravity 'g' of that place. Can you establish a relation between these quantities 'h', 'S', 'r', ' $\rho$ ', 'g' using Dimensional Analysis? If not why? What limitation do you see here?
- Q20) State the limitations of Dimensional Analysis.

\*\*\*FND\*\*\*